Review for Math 1151 Final Exam

1. Find the vertex of the parabola which is the graph of the function $f(x) = -2x^2+8x+3$.

2. A farmer has 5000 yards of fencing and wants to enclose a rectangular plot that borders on a straight river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?

3. Form a polynomial of degree 3 with the zeros -2, 3, and 5.

4. Given the function $f(x) = (x^2 - 9)(x + 4)^2$ list each real zero and its multiplicity.

5. Given the polynomial function $f(x) = x(x + 4)(x - 3)^2$, find the x intercepts of the graph of $f(x)$.

6. Find the vertical and horizontal asymptotes for the rational function

$$R(x) = \frac{3x^2 + 5}{(x + 4)(x - 6)}$$

7. List the potential rational zeros of the polynomial function $f(x) = 5x^4 - x^3 + x^2 + 9$. Do not attempt to find the zeros.

8. Tell the maximum number of zeros for the polynomial function $f(x) = 5x^4+2x^2-6x-5$. Use Descartes' Rule of Signs to determine how many positive and how many negative zeros the polynomial function may have.

9. Find all the real zeros of

$$f(x) = x^3 + 8x^2 + 11x - 20.$$

10. Form a polynomial of degree 4 with real coefficients having 5 as a zero of multiplicity 2, and $3 + 2i$ as a complex root.

11. Use synthetic division to determine if $x - 2$ is a factor of $3x^4 - 6x^3 - 5x + 10$.

12. Find the length of the arc subtended by an angle of 2.5 radians on a circle of radius 6 feet.

13. The measure of an angle is 125°. What is the measure of this angle in radians?

14. The measure of a certain angle is 2.5 radians. What is the measure of this angle in degrees? Give answer as a decimal in degrees.

15. The minute hand of a clock is 15 inches long. How far does the tip of the minute hand move in 25 minutes?

16. Find the exact value of $\tan(8\pi/3)$.

17. Find the exact value of $\sin(210^\circ)$.

18. The point $(-5, 12)$ is on the terminal side of an angle θ (in standard position). Find the exact values of the six trigonometric functions of θ.

19. Suppose $\sin\theta = 5/13$ and θ is in quadrant II, find $\cos\theta$ and $\tan\theta$.

20. Suppose $\tan\theta = 3/4$ and θ is in quadrant III, find $\sin\theta$ and $\cos\theta$.

21. Sketch the graph of $y = 2\tan(3x)$, $-\pi \leq x \leq \pi$, and identify the vertical asymptotes.

22. Determine the amplitude and period of the function $y = -3\cos(x + \pi/2)$.

23. Determine the amplitude, period, and phase shift for the function

$$y = -4\sin(2x + \pi/2).$$

24. Find the exact value of

    (a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

    (b) $\tan^{-1}(-1)$

    (c) $\tan^{-1}(\sqrt{3})$

25. Find the exact value of

    (a) $\tan[\sin^{-1}(-1/2)]$

    (b) $\cos^{-1}[\cos(5\pi/4)]$

    (c) $\sin[\tan^{-1}(-3)]$

26. Establish each identity

    (a) $(\sin\theta)(\cos\theta)(\tan\theta + \cot\theta) = 1$

    (b) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

    (c) $(1 + \sin\theta)(\sec\theta - \tan\theta) = \cos\theta$

    (d) $(\sin\theta)(1 - \cot\theta) + \cos\theta = \sin\theta$

27. Establish each identity

    (a) $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = 2\tan\theta$

    (b) $\dfrac{1 - \cot\theta}{\sin\theta - \cos\theta} = \dfrac{1}{\sin\theta}$

    (c) $\dfrac{(\sec\theta)(\sin^2\theta)}{1 + \sec\theta} = 1 - \cos\theta$

28. Use a trigonometric identity to find the exact value of $\cos(7\pi/12)$.

29. Find the exact value of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ if $\tan\alpha = -4/3$, α in quadrant II, and $\cos\beta = 1/2$, β in quadrant I.

30. Find the exact value of $\sin(\alpha-\beta)$ and $\tan(\alpha+\beta)$ if $\sin\alpha = 5/13$, $\sin\beta = 3/5$, α and β in quadrant I.

31. Find the exact value of

$$\cos[\tan^{-1}(5/12) - \sin^{-1}(-3/5)]$$

32. Find the exact value of $\sin(2\theta)$ and $\cos(2\theta)$ if $\sin\theta = -5/6$ and θ is in quadrant III.

33. Find the exact value of $\sin(\theta/2)$ and $\cos(\theta/2)$ if $\sin\theta = -5/13$ and $180^\circ < \theta < 270^\circ$.

34. Find the exact value of $\sin(2\theta)$ and $\cos(2\theta)$ if $\tan(\theta) = -5/8$ and θ is in the fourth quadrant.

35. Find the exact value of $\sin(\theta/2)$ and $\cos(\theta/2)$ if $\tan(\theta) = -3/4$ and $3\pi/2 < \theta < 2\pi$.

36. Solve each equation on the interval $0 \le \theta < 2\pi$.

    (a) $2\cos(\theta) + 3 = 2$
    (b) $\tan^2\theta = 1/3$
    (c) $2\sin^2\theta - 1 = 0$
    (d) $\cos(2\theta) = -1/2$
    (e) $\sin(2\theta) = \sqrt{2}\cos\theta$

37. Solve each equation on the interval $0 \le \theta < 2\pi$.

    (a) $2\cos^2\theta + \cos\theta = 0$
    (b) $\sin^2\theta - \cos^2\theta = 1 + \cos\theta$
    (c) $(10\cos\theta + 9)(\tan\theta - 5) = 0$
    (d) $\cos(2\theta) + 6\sin^2\theta = 0$

38. A radio transmission tower is 200 feet high. How long should a guy wire be if it is attached to the top of the tower and is to make an angle of 25° with the ground?

39. A ship is just off shore of New York City. A sighting is taken of the Statue of Liberty, which is 305 feet tall. If the angle of elevation to the top of the statue is 20°, how far is the ship from the base of the statue?

40. Find a and α in a triangle if $b = 40$, $c = 30$, and $\beta = 40^\circ$.

41. Find c in a triangle if $a = 40$, $b = 50$, and $\beta = 60^\circ$.

42. In order to find the distance from point A to point B, a surveyor walks off a distance of 400 feet from point A to point C. He measures the angle BAC to be 40° and the angle ACB to be 50°. What is the distance from A to B?

43. An airplane is spotted by two observers who are 1000 feet apart. The angle of elevation of the airplane from observer A is 15° and the angle of elevation from observer B is 20°. The airplane is flying directly above the line through A and B. How high is the airplane?

44. Find the angles in the triangle with sides $a = 40$, $b = 60$, and $c = 30$.

45. Find side b in the triangle with sides $a = 30$ and $c = 20$ and the angle $\beta = 110^\circ$.

46. Find the area of the triangle with sides $a = 50$, $b = 80$, and $c = 90$.

47. Find the rectangular coordinates of each of the following points whose polar coordinates are given:

    (a) $(-3, -\pi/4)$

    (b) $(4, 5\pi/4)$

    (c) $(2, 2\pi/3)$

48. Find three different sets of polar coordinates of each of the following points whose rectangular coordinates are given:

    (a) $(4, 4)$

    (b) $(-\sqrt{3}, 1)$

    (c) $(2, -2)$

49. Solve the system of equations

$$\begin{aligned} (1/2)x + (1/3)y &= 6 \\ (1/4)x - (2/3)y &= -2 \end{aligned}$$

50. Use the method of substitution to find the solution of the system of equations

$$\begin{aligned} 3x - y &= 19 \\ x^2 + y^2 &= 41 \end{aligned}$$

51. A movie theater charges \$10.00 for adults and \$8.00 for senior citizens. One day when 350 people paid an admission, the total receipts were \$3206. How many who paid were adults and how many were seniors?

52. A store sells cashews for \$6.00 per pound and peanuts for \$2.00 per pound. The manager decides to mix some peanuts and cashews to get a mixture that sells for \$3.50 per pound. He wants 72 pounds of the mixture. How many pounds of cashews and how many pounds of peanuts should he use?

53. Solve the following system of equations

$$\begin{aligned} x - y + z &= -4 \\ 2x - 3y + 4z &= -15 \\ 5x + y - 2z &= 12 \end{aligned}$$

54. Find the partial fraction decomposition for the fraction

(a) $\dfrac{2x - 44}{(x + 8)(x - 4)}$

(b) $\dfrac{-8x + 15}{x^2(x + 5)}$

55. Find the partial fraction decomposition for the fraction

$$\frac{9x^2 - 3x - 43}{(x - 4)(x^2 + 9)}$$

56. A sequence is defined recursively by $a_n = n + 2a_{n-1}$, $a_1 = 2$. Write the first five terms of this sequence.

57. Find the sum $\sum_{k=1}^{6}(3k - 7)$.

58. Express the sum $2 - 4 + 8 - 16 + \ldots + (-1)^{10}(2)^{11}$ using summation notation.

59. The n-th term of an arithmetic sequence is $a_n = 3n+5$. What is the common difference?

60. The first term of an arithmetic sequence is 10 and the common difference is 4. Find an expression for the n-th term.

61. Find the sum

$$-9 - 4 + 1 + 6 + \ldots + 496$$

62. Find the fourth and the n-th term of a geometric sequence whose first term is $a = 5$ and common ratio is $r = 2$.

63. Find the sum $\sum_{k=1}^{10} 2\left(\frac{1}{3}\right)^k$

64. Suppose that you have just been hired at an annual salary of \$25,000 and you will receive annual increases of 5%. What will your salary be during your fifth year?

Review for Math 1151 Final Exam – Solutions

1. $(2, f(2)) = (2, 11)$.
2. 3125000 square yards.
3. $f(x) = (x+2)(x-3)(x-5) = x^3 - 6x^2 - x + 30$
4. $x = 3$ (multiplicity 1); $x = -3$ (multiplicity 1); $x = -4$ (multiplicity 2).
5. $(0, 0)$, $(-4, 0)$, and $(3, 0)$.
6. The vertical asymptotes are $x = -4$ and $x = 6$. The horizontal asymptote is $y = 3$.
7. $\pm 1, \pm 3, \pm 9, \pm\frac{1}{5}, \pm\frac{3}{5}, \pm\frac{9}{5}$
8. f has four zeros. f has one positive zero and one negative zero. The other two zeros are complex.
9. $f(x) = (x-1)(x+4)(x+5)$. The real zeros are $x = 1$, $x = -4$, and $x = -5$.
10. $$\begin{aligned}(x-5)^2(x-(3+2i))(x-(3-2i)) &= (x-5)^2(x^2-6x+13)\\ &= (x^2-10x+25)(x^2-6x+13)\\ &= x^4-16x^3+98x^2-280x+325\end{aligned}$$
11. $(x-2)$ is a factor, since $x = 2$ is a root.

$$\begin{array}{r|rrrrr} & 3 & -6 & 0 & -5 & 10 \\ 2 & & 6 & 0 & 0 & -10 \\ \hline & 3 & 0 & 0 & -5 & 0 \end{array}$$

12. $s = r\theta = (2.5)(6 \text{ ft}) = 15 \text{ ft}$.
13. $125^\circ \times \dfrac{\pi}{180} = \dfrac{25\pi}{36}$ radians.
14. $2.5 \times \dfrac{180^\circ}{\pi} = 143.24^\circ$
15. $s = r\theta = (15\text{in}) \cdot \dfrac{25}{60} \cdot 2\pi = 39.27 \text{ in}$.
16. $\tan\dfrac{8\pi}{3} = \tan\dfrac{2\pi}{3} = -\tan\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{3}$
17. $\sin(210^\circ) = \sin(180^\circ + 30^\circ) = -\sin(30^\circ) = -\dfrac{1}{2}$

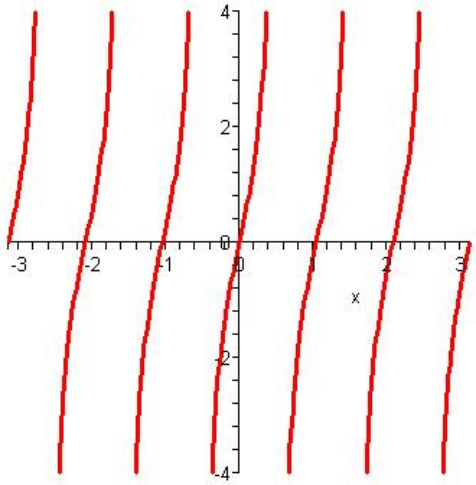
18. $x = -5$, $y = 12$, $r = 13$.
$sin\theta = \dfrac{12}{13}$, $\cos\theta = -\dfrac{5}{13}$, $\tan\theta = -\dfrac{12}{5}$, $\csc\theta = \dfrac{13}{12}$, $\sec\theta = -\dfrac{13}{5}$, $\cot\theta = -\dfrac{5}{12}$.

19. $y = 5$, $r = 13$, $x = 12$.
$\cos\theta = -\dfrac{12}{13}$, $\tan\theta = -\dfrac{5}{12}$.

20. $x = -4$, $y = -3$, $r = 5$.
$\sin\theta = -\dfrac{3}{5}$, $\cos\theta = -\dfrac{4}{5}$.

21. The function has period $\dfrac{\pi}{3}$, and it has vertical asymptotes at

$$x = -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$



22. Amplitude = 3, Period = 2π

23. Amplitude = 4, Period = π, Phase shift = $-\pi/4$

24. (a) $\pi/4$ (b) $-\pi/4$ (c) $\pi/3$

25. (a) $-\sqrt{3}/3$ (b) $3\pi/4$ (c) $-3\sqrt{10}/10$

26. (a) $(\sin\theta)(\cos\theta)(\tan\theta + \cot\theta) = (\sin\theta)(\cos\theta)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right) =$
$(\sin\theta)(\cos\theta)\left(\dfrac{\sin\theta}{\cos\theta}\right) + (\sin\theta)(\cos\theta)\left(\dfrac{\cos\theta}{\sin\theta}\right) = \sin^2\theta + \cos^2\theta = 1$

(b) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta) +$
$(\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta) = (1 + 2\sin\theta\cos\theta) + (1 - 2\sin\theta\cos\theta) = 2$

(c) $(1+\sin\theta)(\sec\theta-\tan\theta) = (1+\sin\theta)\left(\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}\right) =$

$\dfrac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta} = \dfrac{1-\sin^2\theta}{\cos\theta} = \dfrac{\cos^2\theta}{\cos\theta} = \cos\theta$

(d) $(\sin\theta)(1-\cot\theta)+\cos\theta = (\sin\theta)\left(1-\dfrac{\cos\theta}{\sin\theta}\right)+\cos\theta =$

$\sin\theta - \sin\theta\cdot\dfrac{\cos\theta}{\sin\theta}+\cos\theta = \sin\theta-\cos\theta+\cos\theta = \sin\theta$

27. (a) $\dfrac{\sec\theta}{\csc\theta}+\dfrac{\sin\theta}{\cos\theta} = \dfrac{1/(\cos\theta)}{1/(\sin\theta)}+\dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta} = \tan\theta+\tan\theta = 2\tan\theta$

(b) $\dfrac{1-\cot\theta}{\sin\theta-\cos\theta} = \dfrac{1-\dfrac{\cos\theta}{\sin\theta}}{\sin\theta-\cos\theta}\cdot\dfrac{\sin\theta}{\sin\theta} = \dfrac{\sin\theta-\cos\theta}{(\sin\theta-\cos\theta)(\sin\theta)} = \dfrac{1}{\sin\theta}$

(c) $\dfrac{(\sec\theta)(\sin^2\theta)}{1+\sec\theta} = \dfrac{\dfrac{1}{\cos\theta}(\sin^2\theta)}{1+\dfrac{1}{\cos\theta}} = \dfrac{\dfrac{\sin^2\theta}{\cos\theta}}{\dfrac{\cos\theta+1}{\cos\theta}} = \left(\dfrac{\sin^2\theta}{\cos\theta}\right)\left(\dfrac{\cos\theta}{\cos\theta+1}\right) =$

$\dfrac{\sin^2\theta}{\cos\theta+1} = \dfrac{1-\cos^2\theta}{\cos\theta+1} = \dfrac{(1-\cos\theta)(1+\cos\theta)}{1+\cos\theta} = 1-\cos\theta$

28. $\cos\dfrac{7\pi}{12} = \cos\left(\dfrac{\pi}{3}+\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{3}\cos\dfrac{\pi}{4}-\sin\dfrac{\pi}{3}\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}-\sqrt{6}}{4}$

29. $\sin(\alpha+\beta) = \dfrac{4-3\sqrt{3}}{10} \qquad \cos(\alpha+\beta) = \dfrac{-3-4\sqrt{3}}{10}$

30. $\sin(\alpha-\beta) = -\dfrac{16}{65} \qquad \tan(\alpha+\beta) = \dfrac{56}{33}$

31. $\dfrac{33}{65}$

32. $\sin(2\theta) = \dfrac{5\sqrt{11}}{18} \qquad \cos(2\theta) = -\dfrac{7}{18}$

33. $\sin(\theta/2) = \dfrac{5\sqrt{26}}{26} \qquad \cos(\theta/2) = -\dfrac{\sqrt{26}}{26}$

34. $\sin(2\theta) = -\dfrac{80}{89} \qquad \cos(2\theta) = \dfrac{39}{89}$

35. $\sin(\theta/2) = \dfrac{2\sqrt{5}}{5} \qquad \cos(\theta/2) = -\dfrac{\sqrt{5}}{5}$

36. (a) $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

(b) $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

(c) $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

(d) $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

(e) $\theta = \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \dfrac{3\pi}{2}$

37. (a) $\theta = \dfrac{\pi}{2}, \dfrac{\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}$

(b) $\theta = \dfrac{\pi}{2}, \dfrac{\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}$

(c) $\theta = \cos^{-1}(-9/10), \quad 2\pi - \cos^{-1}(-9/10), \quad \tan^{-1} 5, \quad \pi + \tan^{-1} 5$
$\theta = 1.373, \quad 2.691, \quad 3.593, \quad 4.515$

(d) No solutions

38. $200/\sin(25^\circ) = 473$ feet

39. $305/\tan(20^\circ) = 838$ feet

40. $\alpha = 111.2^\circ \qquad a = 58.0$

41. $c = 56.1$

42. $|AB| = 400 \cos 40^\circ = 306.4$ feet

43. $h = 1000/(\cot 15^\circ - \cot 20^\circ) = 1016$ feet

44. $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{29}{36} \qquad \alpha = \cos^{-1}(29/36) = 36.3^\circ$
$\cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac} = -\dfrac{11}{24} \qquad \beta = \cos^{-1}(-11/24) = 117.3^\circ$
$\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{43}{48} \qquad \gamma = \cos^{-1}(43/48) = 26.4^\circ$

45. $b^2 = a^2 + c^2 - 2ac\cos\beta = 1710 \qquad b = \sqrt{1710} = 41.4$

46. $s = (50 + 80 + 90)/2 = 110$, $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{110 \cdot 60 \cdot 30 \cdot 20} = 600\sqrt{11} = 1989.97$

47. (a) $\langle -3, -\pi/4\rangle = (-3\cos(-\pi/4), -3\sin(-\pi/4)) = (-3\sqrt{2}/2, 3\sqrt{2}/2)$
(b) $\langle 4, 5\pi/4\rangle = (4\cos(5\pi/4), 4\sin(5\pi/4)) = (-2\sqrt{2}, -2\sqrt{2})$
(c) $\langle 2, 2\pi/3\rangle = (2\cos(2\pi/3), 2\sin(2\pi/3)) = (-1, \sqrt{3})$

48. (a) $(4, 4) = \langle 4\sqrt{2}, \pi/4\rangle = \langle 4\sqrt{2}, 9\pi/4\rangle = \langle -4\sqrt{2}, 5\pi/4\rangle$
(b) $(-\sqrt{3}, 1) = \langle 2, 5\pi/6\rangle = \langle -2, -\pi/6\rangle = \langle -2, 11\pi/6\rangle$
(c) $(2, -2) = \langle 2\sqrt{2}, -\pi/4\rangle = \langle -2\sqrt{2}, 3\pi/4\rangle = 2\sqrt{2}, 7\pi/4\rangle$

49. $(8, 6)$

50. $(5, -4)$ or $(6.4, -0.2)$

51. $A + S = 350$, $10A + 8S = 3206$; $A = 203$, $S = 147$. There were 203 adults and 147 seniors.

52. $P + C = 72$, $2P + 6C = 3.5 \cdot 72$; $P = 45$, $C = 27$. He should use 45 pounds of peanuts and 27 pounds of cashews.

53. $(1, 3, -2)$

54. (a) $\dfrac{5}{x+8} - \dfrac{3}{x-4}$

    (b) $\dfrac{11}{5(x+5)} - \dfrac{11}{5x} + \dfrac{3}{x^2}$

55. $\dfrac{136x + 469}{25(x^2+9)} + \dfrac{89}{25(x-4)}$

56. 2, 6, 15, 34, 73

57. $(-4) + (-1) + 2 + 5 + 8 + 11 = 21$

58. $\sum_{k=1}^{11} (-1)^{k-1} 2^k$

59. $d = a_1 - a_0 = 11 - 8 = 3$

60. $a_n = 10 + 4(n-1) = 6 + 4n$

61. $n = \dfrac{a_n - a_1}{d} + 1 = \dfrac{496 - (-9)}{5} + 1 = 102$; $S = \dfrac{n}{2}(a_1 + a_n) = \dfrac{102}{2}(-9 + 496) = 24837$

62. $a_n = 5 \cdot 2^{n-1}$; $a_4 = 5 \cdot 2^3 = 40$

63. $\displaystyle\sum_{k=1}^{10} 2\left(\frac{1}{3}\right)^k = a\,\frac{1 - r^n}{1 - r} = \frac{2}{3}\frac{1 - (1/3)^{10}}{1 - (1/3)} = 1 - (1/3)^{10}$

64. $A = P(1 + r)^n = 25000(1 + .05)^4 = \$30387.66$