# Lesson 6: Time Value of Money (TVM)

We will study the following topics in this lesson.

1. Simple interest
2. Compound interest
3. Frequency of compounding
4. Effective and nominal interest rates
5. Cash inflows and outflows
6. Present and future value
7. Loans, annuities, and sinking funds

## *Simple interest*

When a bank lends money, the borrower must pay back the original amount he borrowed plus interest. The original amount is called the **principal**. The total amount repaid, including principal and interest, is called the **maturity value** or **accumulated value.**

The amount of interest is determined by three quantities – the **principal**, the **rate of interest**, and the **duration (time)**. These quantities are related by the following formula.

Interest = Principal x Rate x Time

$$\mathbf{I = P\ r\ t}$$

The interest rate is usually expressed as a percentage, but it must be converted to a decimal when using this formula. For example, 9% should be converted to 0.09.

The maturity value of this loan is given by:

Maturity Value = Principal + interest

$$\mathbf{A = P + I}$$

**Example 1.** Janet applied for a 3 year loan of $75,000 from a bank. The bank granted the loan with an annual interest rate of 13.5%. (a) What is the simple interest on the loan? (b) What is the maturity value of the loan?

**Solution:** First, identify the variables..

P = 75,000
r = 13.5% = 0.135
t = 3

Substituting into the simple interest formula gives I = ($75,000)(0.135)(3) = $30,375.

The maturity value of this loan, or the total amount to be repaid, is

A = $75,000 + $30,375 = $105,375.

Although the time span of a loan may be given in any length – days, months or years – the rate of interest is an annual rate. Therefore when the duration of the loan is given in months or days, the time must be converted to years. For example, when the time is given in months, the following step is required.

$$t = \frac{\text{number of months}}{12}$$

**Example 2**. Find the simple interest on Rodney's loan of $7,500 if the loan has a rate of 14% and is due in 18 months. Also, what is the maturity value of this loan?

**Solution:** We have Principal = $7,500, Rate = 14%, and Time = 18 months = 1.5 years. Substituting these values we arrive at I = ($7,500)(.14)(1.5) = **$1,575.00.**

The maturity value of this loan is: A = $7,500.00 + $1,575.00 = **$9,075.00.** This is the total amount required to pay back at the end of the term of the loan.

**Example 3.** $5,000 is invested in an account earning simple interest at an annual rate of 5%. How much interest is earned after 4 years?

**Solution:** I = ($5,000)(.05)(4) = $1,000.

## *Compound interest*

Compound interest is interest that is computed on the accumulated interest as well as on the original principal. Compound interest is involved when you leave money in a savings account for more than one interest period, or when you borrow money for more than one interest period.

**Example 4.** Suppose that you invest $1,000 in an account earning 10% interest, and leave the money in the account for a number of years. What is the maturity value after 5 years?

- In the first year, the account will earn interest of $100 (10% of $1,000), and the accumulated value at the end of the first year will be $1,000 + $100 = $1,100.
- In the second year, the account will earn interest of $110 (10% of $1,100). This represents $100 earned on interest from the original principal, plus $10 earned on the accumulated interest. The accumulated value at the end of the second year will be $1,100 + $110 = $1,210.
- In the third year, the account will earn interest of $121 (10% of $1,210). This represents $100 earned on interest from the original principal, plus $21 earned on

the accumulated interest.

| Year | Amount at start of year | Interest earned | Amount at end of year |
|---|---|---|---|
| 1 | $1,000.00 | $100.00 | $1,100.00 |
| 2 | $1,100.00 | $110.00 | $1,210.00 |
| 3 | $1,210.00 | $121.00 | $1,330.00 |
| 4 | $1,330.00 | $133.10 | $1,464.10 |
| 5 | $1,464.10 | $146.41 | $1,610.51 |

Note that the value at the end of each year is 10% greater than the value at the end of the previous year; that is, it is 1.1 times greater. In order to calculate the amount at the end of the fifth year, we start with $1,000 (the principal) and multiply by 1.1 five times. In other words, we compute $\$1,000 * (1.1)^5$.

Hopefully, it is clear how to extend the idea to different interest rates and terms. Suppose that we invest an amount P (the principal) at an interest rate r, for n years. The amount A in the account is initially equal to P, but each year it is multiplied by (1+r). At the end of n years, the accumulated value A is equal to $P * (1+r)^n$. This is called the compound interest formula, and it is very important, so we will put a box around it.

$$\boxed{A = P\,(1 + r)^n}$$

**Example 5.** Joe borrows $2,500 at 12% annual interest. If he makes no payments, how much will he owe after 5 years? How much interest will he have accumulated?

**Solution:**

$$A = P(1 + r)^n = \$2{,}500\,(1 + .12)^5 = \$4{,}405.85$$

$$I = A - P = \$4{,}405.85 - \$2{,}500.00 = \$1{,}905.85$$

Joe will owe $4,405.85, including the original principal of $2,500.00, and $1,905.85 in accumulated interest.

## *Frequency of compounding*

In the above examples, interest is only computed once per year – we say that interest is compounded annually. However, interest is often compounded more frequently than this. The number of times per year that interest is compounded is called the frequency of compounding. We use the letter *t* to denote the frequency of compounding. Some common values of *t* are listed below.

- t = 1 : annual compounding
- t = 2 : semi-annual compounding
- t = 4 : quarterly compounding
- t = 12 : monthly compounding
- t = 365 : daily compounding (sometimes t = 360 is used)

If interest is compounded t times per year, then we must divide the annual interest rate r by t to get the periodic interest rate r/t. Also, we must multiply the number of years n by t to get the number of periods nt. Our revised compound interest formula is shown below.

$$A = P\,(1 + r/t)^{nt}$$

The more frequently interest is compounded, the faster interest will accumulate, as the next example shows.

**Example 6.** Janet borrows $10,000 for three years at an annual interest rate of 6%. Find the maturity value after 3 years if interest is compounded (a) annually, (b) monthly, or (c) daily.

**Solution:**

(a) $A = \$10{,}000 * (1 + .06)^3 = \$11{,}910.16$

(b) $A = \$10{,}000 * (1 + .06/12)^{36} = \$11{,}966.81$

(c) $A = \$10{,}000 * (1 + .06/365)^{3\times 365} = \$11{,}972.00$

## *Effective annual rate*

The effective annual rate is used to compare interest rates with different frequencies of compounding – it reflects the true cost of borrowing. A nominal interest rate *r* compounded *t* times per year, is equivalent to a higher effective interest rate R compounded annually. The relationship between the two rates is given by the following formula.

$$R = (1 + r/t)^t - 1$$

**Example 7.** A credit card charges 15.5% interest, compounded monthly. A second credit card charges 16.5% interest, compounded annually. Which credit card has a better rate?

**Solution:** The effective annual rate of the first card is $R = (1 + 0.155/12)^{12} - 1 = 0.1665 = 16.65\%$. This is higher than 16.5%, so the second credit card has a better rate.

## *TVM Variables*

In this section we assume that the reader has a TI-83 Plus calculator, but the procedure will be similar for other calculators with financial functions. Press [Apps] [Enter] [Enter] to start the TVM Solver. (TVM stands for "Time Value of Money." This displays a worksheet with seven variables.

- N      number of payment periods (**not necessarily years)**
- I%    interest rate
- PV    present value (or principal)
- FV    future value (or maturity value)
- PMT payment amount (each period)
- P/Y   number of payment periods per year
- C/Y   number of compounding periods per year (t)

The last line indicates whether payments are made at the beginning or the end of a period. End of period is the default setting. I do **not** recommend changing this setting, because it is easy to forget to change it back.

There are several important details to bear in mind. The number of payment periods is not always the same as the number of years – if payments are made monthly, then N will denote the number of months. The interest rate I% is a percentage, not a decimal; thus an interest rate of 5% would be entered as 5 rather than as 0.05. Finally, the values of PV, FV and PMT may be positive or negative, depending on whether they represent inflows or outflows of cash.

Note carefully that the value of P/Y will affect the final answer, even if no payments are made. If periodic payments are not made (PMT = 0) then P/Y should be the same as C/Y, and N should be equal the number of compounding periods over the term. For example, if interest is compounded monthly for 3 years, then P/Y and C/Y should both be set to 12; and N should be set to 36, since there are 36 months in the term. In any case, remember that N must equal (P/Y) times the number of years.

The TVM solver assumes that equal payments are made at equal intervals. If the payments vary or are not made at equal intervals, then life gets a lot more complicated. We do not worry about this complication in this course.

## *Cash flows*

When doing financial calculations, it is important to distinguish between inflows and outflows of cash. An outflow occurs when you spend or invest money, or make a payment. An inflow occurs when you receive, withdraw or borrow money.

**Example 8.** Amy borrows $6,000 to buy a new car: **PV = +6000.** When Amy borrows the money, she receives $6,000 in cash from the bank, which she uses to purchase a car. This is an **inflow** because she receives money from the bank, and it is a **present value** because she receives the money when the account is opened.

**Example 9.** Bruce opens a savings account with an initial deposits of $385: **PV = –385.** This is an **outflow** of cash because he is giving cash to the bank.

**Example 10.** Coraline saves money to purchase a $3600 entertainment system: **FV = +3600.** This is an inflow of cash because she receives (withdraws) $3600 from the bank, and it is a **future value** because the cash is received upon closing the account.

**Example 11**. Daryl makes a mortgage payment of $1155: **PMT = –1155.** Daryl must give cash to the bank to make the payment, so it is an outflow.

**Example 12.** Edith receives an interest payment of $30: **PMT = +30.** Edith receives money from the bank, so it is an inflow.

## *Loan amortization, sinking funds and annuities*

### Annuities

An ***annuity*** is a series of equal periodic payments or deposits with the interest on each one being compounded interest. An example of an annuity would be the lottery. It is a **steady stream of payments**.

The compound amount of each payment is computed and the amount of the annuity is the sum of the compound amounts of **each** payment. For example: if you deposited $50.00 each month for one year, at the end of the year you would have varying lengths of time that each payment of $50.00 was earning interest. The $50.00 deposited in January would have earned 12 months interest at the end of December, the deposit made in February would have earned 11 months interest and so on.

The time between each payment is called the ***payment interval***, and the time from the beginning of the first payment interval to the end of the last one is called the ***term of the annuity***.

There are two types of annuities: ***ordinary annuities*** and ***annuities due.*** They differ only in when the payments are made. In an ordinary annuity, the payments are made at the end of each period. In an annuity due, payments are made at the beginning of each period. We only consider the first type of annuity in this course.

## Sinking Funds

A *sinking fund* is a financial arrangement that sets aside regular periodic payments of a particular amount of money. Interest accumulates the same as any compounding account but to a specific pre-determined amount [future value] at a known future date. It is primarily used in business by large corporations to discharge bond indebtedness, to replace worn-out equipment for plant expansion, and so on. Also, if an individual makes regular monthly deposits in an interest-bearing account in order to save for an anticipated purchase, then this can be considered a sinking fund.

## Loan amortization

Amortization is the process of liquidating a debt by paying in equal installments (usually monthly). Car loans and mortgages are common examples of amortized loans. In loan amortization problems, the amount borrowed is the present value, and we are generally interested in calculating the periodic payment. The future value is usually zero, because the last payment retires the debt and closes the account. (A negative FV would signify a ***balloon payment***.)

# *Solving TVM problems with the TI-83 Plus calculator*

**Example 13.** $5,000 is invested in an account earning 5% annual interest. What will the value of the account be in 4 years? How much interest will have been earned?

**Solution:**

There are no payments, so PMT = 0. The interest rate is 5%, so I% = 5. Interest is compounded annually, so C/Y = 1 and P/Y = 1. The number of payment periods is N = 4 (years). The present value is –5000, because deposits are outflows.

| | | | |
|---|---|---|---|
| N = 4 | I% = 5 | PV = –5000 | PMT = 0 |
| **FV = +6077.53** | P/Y = 1 | C/Y = 1 | |

Using the TVM Solver, we find that the maturity value is $6,077.53. Since the principal is $5,000, the amount of interest earned is $1,077.53.

**Example 14.** Irene borrows $1,250 at an annual interest rate of 6%, compounded monthly. If she makes no payments, how much will she owe in 2.5 years?

**Solution:**

| | | | |
|---|---|---|---|
| N = 30 | I% = 6 | PV = +1250 | PMT = 0 |
| **FV = –1451.75** | P/Y = 12 | C/Y = 12 | |

PMT = 0 because no payments are made. C/Y = P/Y = 12 because interest is compounded 12 times per year. PV is the principal, and it is positive because it is an inflow of cash for Irene. The maturity value is FV = –1451.75; it is negative because Irene must pay this amount to the bank to close the account.

**Example 15.** Gina is 40 years old and she has accumulated $50,000 in a savings account. She adds $100 at the end of each month to an account that pays an annual interest rate of 6%, compounded quarterly. Will she be able to retire in 20 years?

**Solution:**

| | | | |
|---|---|---|---|
| N = 240 | I% = 6 | PV = –50000 | PMT = 0 |
| **FV = +210574.70** | P/Y = 12 | C/Y = 4 | |

Did you expect PV to be negative? We pretend that Gina is opening the account today. This requires an initial deposit of $50,000, and a deposit is an outflow of cash. P/Y = 12 since payments are made monthly, and the payment amount is PMT = –100. However, C/Y = 4 since the interest is compounded quarterly. The number of payment periods is 240, since Gina makes 12 payments per year for 20 years. ($12 \times 20 = 240$)

Using the TVM Solver, we find that she will have accumulated $210,574.70 after 20 years.

**Example 16.** Kara receives $15,000 per year for 12 years. If the interest rate is 6% per year, what is the value of this annuity?

**Solution:**

| | | | |
|---|---|---|---|
| N = 12 | I% = 6 | **PV = –125757.66** | PMT = +15000 |
| FV = 0 | P/Y = 1 | C/Y = 1 | |

It might be surprising that PV is negative, but it makes sense if you think about it. The solver is telling us that, in order to generate an income of $15,000 per year for 12 years, Kara would need to invest $125,757.66. An investment is a cash outflow, and this is why we get a negative number.

**Example 17.** Leon gets a 30-year mortgage for a $190,000 home. The interest rate is 4.8%, compounded monthly. What is the monthly payment?

| | | | |
|---|---|---|---|
| N = 360 | I% = 4.8 | PV = +190,000 | **PMT = –996.86** |
| FV = 0 | P/Y = 12 | C/Y = 12 | |

**Solution**:

There are 360 monthly payments. The present value is positive, because borrowing money represents a cash inflow. On the other hand, the monthly payments of $996.86 are cash outflows.

**Example 18.** Hector borrows $28,000 to purchase a new car. The interest rate is 7%, compounded daily, and the term of the loan is 48 months. What is the monthly payment?

| | | | |
|---|---|---|---|
| N = 48 | I% = 7 | PV = +28000 | **PMT = –670.75** |
| FV = 0 | P/Y = 12 | C/Y = 365 | |

# Assignment

*Show all work for each problem. If you are using the TVM solver, then write out the values of the seven TVM variables, as well as your final answer.*

1. Aaron borrows $7000 at 6% simple annual interest for 15 months. What is the interest charge?
2. Brianna borrows $2000 at 6.5% interest, compounded annually. If she makes no payments, how much will she owe in 3 years?
3. Calvin invests $700 in a savings account earning 3% annual interest, compounded monthly. How much will his investment be worth in 27 months?
4. Delia wishes to save $2500 for a vacation in 16 months. How much should she save at the end of each month to achieve her goal, if her savings account earns 4% annual interest?
5. Emmett receives an annuity of $9,600 per year for 8 years. What is the present value of this annuity, assuming an annual interest rate of 6%?
6. Fiona borrows $175,000 to purchase a house. Assume that she takes out a 25-year mortgage at 5.75% interest, compounded monthly. What is her monthly payment?
7. Galvin owes $4,200 in credit card debt. The interest rate is 16%. If he pays $300 each month, how long will it be until his debt is retired?
8. A savings account has an interest rate of 2.5%, compounded monthly. What is the effective annual rate?